1. INTRODUCTION

A weakly/unconsolidated formation is inevitably damaged during the process of well completion, and thereafter degraded/eroded during hydrocarbon recovery. The resulting sanding problem (sand production) causes wear of production equipment and creates disposal requirements for an environmentally acceptable manner. On the other hand, the hydrocarbon rate might yet be enhanced by several folds increase in sand-free rates after a certain amount of sand is produced. The production of sand has been proven to be an effective way to increase well productivity both in heavy oil and conventional reservoirs. Is there an economic balance to maximum oil production and minimum sand? Before we can answer this question, it is vital to understand the mechanisms of sand fluidization and develop an efficient computational model that can be used to predict volumetric sand production and associated wellbore stability prior to or during field operations. The ability to predict accurately sanding tendencies and volumetric production improves not only completion designs (completion types, orientation, perforation sizes, etc.), but also facilitates workover planning and reduces the operational cost.

Sand production mechanisms can be summarized into the following points:

- Shear failure induced by fluid pressure drawdown can lead to the breaking of sand grain bonds and the alteration of the material’s mechanical properties;
- Tensile failure caused by high hydrocarbon production rates can lead to dilation of solid skeleton and the loss of solid particles mechanical interactions through disaggregation;
- High stresses due to completion cause the formation to fail (in compression) whereas fluid...
viscous drag forces bring the failed materials from the perforation tunnels into the wellbore.

The numerical modeling of sand fluidization is a challenging task since it involves capturing the whole range of material response from bulk to fluid like behaviour, which is strongly controlled by a combination of stress changes and fluid flow in relation to the strength properties of sand matrix. The discrete element method seems to lend itself ideally to the computation of such physical phenomenon. However, this method is limited since it requires, among others, exorbitant computer power, precise description of contact behavior between granular particles, and rigorous incorporation of the fluid phases. Hence, a continuum mechanics approach is still a powerful alternative-modeling framework in which sand production can be treated as an erosion process. More precisely, the detachment of sand particles is driven by the interaction of hydrodynamics and geomechanics, see Figure 1. The hydro-erosion model, first proposed by Vardoulakis et al. [1], is based on rigid porous media (no skeleton deformation) in which mass balance is applied to a three-constituent system comprised of solid, fluid and fluidized solid using homogenization mixture theory. Subsequently, Wan and Wang [2,3,4] extended this pure erosion model to include the effect of the deformation of porous media in a consistent manner, where a single-phase flow is fully coupled with geomechanics within a continuum mechanics framework. Furthermore, Wang et al. [5] extended previous work to a fully coupled reservoir-geomechanics model to account for the effects of multiphase flow and geomechanics as well as their interaction in a consistent manner. The model has been proven to be a viable tool in terms of matching numerical calculations with lab test and field data [6]. As the pressure drawdown and reservoir depletion takes place, the erosion process begins as a result of the degradation of the sand matrix strength and the drag force imposed by fluid pressure gradient. The plastic yielding zones develop due to the material degradation (erosion) and stress re-distribution, while the wormholes or cavities form and propagate in terms of the increasing porosity values. The volumetric oil and sand productions are also calculated as a function of time, stresses, and hydrocarbon flow rate. This approach results in solving a set of coupled non-linear time-dependent equations with fluidized solid concentration (saturation), fluid pressure, porosity, and deformation as main variables.

In this paper, we study an oilwell subjected to two different completion schemes (openhole and perforated casing completions) to predict the wellbore stability and related volumetric sand as well as oil production as a function of time, stress, fluid properties, and completion schemes. This study is intended to initiate the first step towards helping oil operators to design an optimized pumping scheme as well as a wellbore completion strategy in terms of maximizing the oil production, while also reducing the risk of well failure and controlling sand production.

2. GOVERNING EQUATIONS FOR COUPLED MULTIPHASE FLOW AND GEOMECHANICS

The fully coupled reservoir and geomechanics sand production model was derived within the same framework of mixture theory in [5], which fully accounted for the effects of multiphase flow of three components (gas, water, oil) and their interaction with geomechanics. For brevity, the fluid/gas saturated sand body is idealized as a Representative Elementary Volume (REV) which comprises of five phases, namely solid grains (s), fluidized solids (fs), fluid (f), water (w) and gas (g) as shown in Figure 2.
The set of governing equations can be summarized as in the following,

\[-\frac{\partial \phi}{\partial t} + \nabla \cdot \left[(1 - \phi) \mathbf{u}_s\right] + \lambda (1 - \phi) S_{fs} \nabla \phi = 0. \quad (1)\]

\[\frac{\partial (\phi (S_{fs} - 1))}{\partial t} + \nabla \cdot \left[S_{fs} \mathbf{v}_m + (1 - \phi + S_{fs}) \mathbf{u}_s\right] = 0. \quad (2)\]

\[\nabla \cdot \left[\frac{\mathbf{v}_s}{B_o} + \frac{S_o \phi}{B_o} \mathbf{u}_s\right] + \frac{\partial}{\partial t} \left[\frac{\phi S_o}{B_o}\right] = 0. \quad (3)\]

\[\nabla \cdot \left[\frac{\mathbf{v}_w}{B_w} + \frac{S_w \phi}{B_w} \mathbf{u}_s\right] + \frac{\partial}{\partial t} \left[\frac{\phi S_w}{B_w}\right] = 0. \quad (4)\]

\[\nabla \cdot \left[\frac{\mathbf{v}_g}{B_g} + \frac{S_g \phi}{B_g} \mathbf{u}_s\right] + \frac{\partial}{\partial t} \left[\frac{\phi R S_o}{B_o} + \frac{\phi S_g}{B_g}\right] = 0. \quad (5)\]

\[\nabla \cdot \left(\sigma^{ef} - \omega \sigma_{m} \mathbf{1} \right) + \mathbf{b} = 0. \quad (6)\]

where \(\phi\) = porosity, \(S_o\) = saturations, \(B_i\) = the formation volume factors, \(\rho_i\) = the densities at stock tank condition \((i=g, w, f, s)\), \(R_i\) = the solution gas oil ratio, \(\sigma^{ef}\) = effective stress, \(\mathbf{b}\) = body forces per unit volume, and \(\omega\) is Biot coefficient accounting for the compressibility of the sand grains. \(\mathbf{1}\) is the Kronecker delta tensor such that \(\delta_{ij}\). The averaged mixture pressure can be defined as \(P_m = S_o P_o + S_g P_g + S_w P_w\). Full details of derivations can be found in Wang et al. [5]. The erosion coefficient \(\lambda\) provides a length scale that can be linked to the accumulated plastic strains \(\gamma^p\) through the following relationship, i.e.

\[\lambda = \lambda(\gamma^p) = \lambda_0 + \frac{\gamma^p}{\gamma^p_{\text{max}}} \quad (7)\]

where \(\alpha\) and \(\beta\) are constants to be determined, while \(\lambda_0\) is a constant, and \(\gamma^p_{\text{max}}\) corresponds to the maximum plastic shear strain calculated for the entire domain. As for describing fluid flow, Darcy's law is used to establish the relation between pressure gradient \(\nabla p_j\) and discharge velocity \(\mathbf{v}_j\) for phases and the mixture \((j=o, g, w, m)\). Thus

\[\mathbf{v}_j = -\frac{k}{\mu} \nabla (p_j + \rho_j g) \quad (8)\]

where \(g\) is the vector of gravitational acceleration, \(k\) is the permeability tensor that can be related to porosity via the variation of Carman-Kozney equation, i.e.

\[k = k_0 \exp \left[A \frac{\phi - \phi_0}{\phi_{\text{max}} - \phi_0}\right] \mathbf{1} \quad (9)\]

where \(k_0\), \(\phi_0\) are constants, and \(A\) is a fitting parameter.

Turning to solid skeleton deformations, a plasticity based constitutive law incorporating dilatancy together with Drucker-Prager failure criterion is used to describe the behavior of the solid skeleton dominated by grain slippage, rearrangement, dilation and de-structuration under compression and shear. The flow rule basically defines the plastic strain increment vector as the normal to the plastic potential function \(G\) and its magnitude is determined from the plastic multiplier \(\Lambda\), i.e.,

\[d\sigma^p = d\Lambda \frac{\partial G}{\partial \sigma^{ef}} \quad (10)\]

\[d\Lambda \geq 0 \quad \text{if} \quad F(\sigma) = 0 \quad \text{and} \quad dF = 0 \]

\[d\Lambda = 0 \quad \text{if} \quad F(\sigma) = 0 \quad \text{and} \quad dF < 0 \quad (11)\]

where \(F\) is a function of stress representing a cone shape in the stress space [7]. In order to describe the internal damage due to the degradation of the porous medium as erosion proceeds, it is assumed that the material properties such as cohesion \(C\) and friction angle \(\varphi\) drop linearly with porosity \(\phi\), i.e.

\[C = C_r + C_0 \frac{\phi_{\text{max}} - \phi}{\phi_{\text{max}} - \phi_0} \quad \varphi = \varphi_r + \varphi_0 \frac{\phi_{\text{max}} - \phi}{\phi_{\text{max}} - \phi_0} \quad (12)\]

where \(C_0\) and \(\varphi_0\) are constants related to initial cohesion and friction angle respectively, and \(\phi_{\text{max}}\) is the maximum porosity at residual friction angle and cohesion, i.e. \(C_r\) and \(\varphi_r\). The friction and cohesion softening as described in Eq. (12) lead to the stress-strain and strain-volumetric strain curves illustrated in Figure 3 for the case of triaxial stress conditions. It is seen that the sand matrix behaves linear elastically before peak strength, and thereafter gradually weakens towards residual strength. The associated volumetric strain curve also shows initial volume compaction followed by dilation.
3. NUMERICAL STRATEGY – COUPLING TECHNIQUES

In order to effectively take advantage of the current advanced standard reservoir and stress-strain model, a modular approach is adopted to incorporate this stress-flow-erosion model. More precisely, this model is implemented into three integrated computational modules, i.e. erosion module, reservoir module, and geomechanics module. The key idea in the modular system is the reformulation of the stress-flow-erosion coupling so that any existing advanced stress and reservoir code can be incorporated with minimum development efforts. This is also termed as a partially coupled approach because the stress, flow and erosion equations are solved separately for each time increment, and the data are passed among them or iterated until convergence is achieved on a time step basis, see Figure 4. The system is powerful in terms of its capabilities, yet practical in terms of computer requirements. Depending on the complexity of addressed problems, different coupling methods can be used, i.e. decoupled, explicitly coupled, iteratively coupled, and fully coupled [8,9].

Decoupled: The flow part of the coupled system does not receive coupled terms back from the deformation and erosion modules, but only supplies incremental changes in pressure, temperature, flux, etc. to them. This is also called one way coupling. The changes calculated in the deformation and erosion modules do not affect the flow solution.

Explicitly coupled: This can be achieved by lagging the coupling terms one time step behind. Based on the previous stress solution, the reservoir simulator calculates the compressibility coefficient and stress-dependent flow properties, and seeks for the flow solution. Then the stress solution is advanced to current time based on the calculated flow solution.

Iteratively coupled: The iterative method consists of the repeated solutions of the flow and stress equations during the time step until certain convergence criteria are satisfied. It is noted that the explicit coupling is a special case of the iteratively coupled system in which only one iteration is performed per time step.

Fully coupled: The fully coupled approach solves the coupled equations simultaneously. Ideally, it has the most advantages, but it needs large code development and difficulties of future maintenance. It is worth to mention that the iteratively coupled method solves the problem as rigorously as a fully coupled (simultaneous) solution if iterated to full convergence.

Hence, the degree of coupling among reservoir, erosion and geomechanics can be modulated at will depending on the level of computations ranging from independent flow and stress solutions to a fully coupled system. The data passing among the modules is described as follows.

Based on the previous stress solution, the reservoir simulator calculates the compressibility coefficient and stress-dependent flow properties. Due to the flow pressure load, the geomechanics module calculates the volumetric changes, which can be related to porosity changes into the reservoir module. The pore volume change is an important recovery mechanism accounting for a large part of the production. Another effect is the stress dependence

Coupling between reservoir and geomechanics: This can be done through pore volume changes and stress-dependent flow properties. Due to the flow pressure load, the geomechanics module calculates the volumetric changes, which can be related to porosity changes into the reservoir module. The pore volume change is an important recovery mechanism accounting for a large part of the production. Another effect is the stress dependence
of permeability [10], as the stress redistribution might alter the reservoir permeability as a result of isotropic expansion/compression and/or the creations of shear planes.

**Coupling between erosion and reservoir:** Porosity increases due to erosion enhance permeability, as defined porosity depended permeability in Eq. (9). In return, erosion activity intensifies with an increase of flow flux as the pressure distribution is modified.

**Coupling between geomechanics and erosion:** Plastic shear deformations, incurred in the solid matrix under fluid and stress gradients, increase the solid erosion potential in Eq. (7). In return, the erosion process weakens the solid matrix through degradation of its mechanical strength, see Eq. (12).

4. NUMERICAL EXAMPLES

Well completions are designed to yield a maximum overall profitability from a reservoir. The idea is to design a lowest cost completion scheme based on the reservoir in situ conditions. This means that the engineer must be able to predict the sanding tendencies and possible volumetric sand production during the span of the completions use. In the following simulation, a numerical example of a light oil reservoir in North Sea is examined under hydrodynamics and geomechanics in both open-hole completion and perforated casing completion. Water and gas effects will be addressed in other publication due to the limitation of space.

![Fig. 5 Mesh layout near wellbore showing perforations.](image)

Figure 5 shows a close-up of the 2-D finite element mesh representing one quarter of a plan section (1m thickness) of a vertical perforated casing well of inner radius $r_0 = 0.1m$ with the outer boundary of the well extending to 5m. The initial fluidized sand saturation $S_{f0}$ and porosity $\phi_0$ are chosen to be 0.0001 and 0.16 respectively. The simulation is conducted as follows. First, the initial state of the reservoir is computed based on an oil pressure of 24.6 MPa and an external stress of 38 MPa is imposed on both inner (including wellbore and perforations) and outer boundaries. Then, the stress around wellbore is changed to a reservoir pressure of 24.6 MPa to simulate the open-hole completion. Finally, a 21.0 MPa fluid pressure is applied (3.6 MPa drawdown) at three perforations ($P_1$, $P_2$, and $P_3$) as shown in Figure 5.

The length of each perforation is 0.25 m with a 0.012m diameter for $P_2$, and a 0.006m diameter for both $P_1$ and $P_3$. These, in fact, refer to eight perforated slots instead of cylindrical perforations for the full 2D well configuration. The initial erosion coefficient is set to 1.8 m$^{-1}$ for the whole domain of the reservoir formation. Finally, the entire finite element grid is comprised of 7541 nodes and 7352 four-noded elements and the time step size used in the analysis is 0.01 day for a total time span of 5 days investigated. Table 1 shows the material properties (fluid and geomechanics) used in the simulation. It is worth to mention this example is different with the one in [5] in which the perforations are treated as highly damaged zones (high initial porosity).

In the open-hole completion analysis, the perforations are filled with finite element meshes and a 3.6 MPa drawdown is applied at the inner boundary of the wellbore. All the properties are the same as perforated casing case. The aim of this study is to examine the difference of volumetric sand production and associated wellbore stability under different completion schemes.

<table>
<thead>
<tr>
<th>Table 1 Model parameters</th>
<th>$\lambda_0 = 1.8$ m$^{-1}$</th>
<th>$\rho_s = 2.67$ g/cm$^3$</th>
<th>$\rho_o = 0.85$ g/cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{0x} = 0.15$ Darcy</td>
<td>$K_{0y} = 0.15$ Darcy</td>
<td>$\mu = 5$ cp</td>
<td></td>
</tr>
<tr>
<td>$C_0 = 6$ MPa</td>
<td>$E = 2$ GPa</td>
<td>$\nu = 0.25$</td>
<td></td>
</tr>
<tr>
<td>$\phi_0 = 38^o$</td>
<td>$\sigma_{ex} = 38$ MPa</td>
<td>$P_0 = 24.6$ MPa</td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.006$</td>
<td>$\beta = 0.2$</td>
<td>$\phi_r = 28^o$</td>
<td></td>
</tr>
</tbody>
</table>

For the purpose of clarity of illustration, the figures are plotted in the vicinity of the wellbore, within the first 0.5m, 1m, 2m and 5m as indicated in $XY$ axes respectively.
4.1. Shear stress and yielding after open-hole completion (before drawdown)

In order to examine the wellbore stability and sand production, it is essential to understand the open-hole completion and perforation process. The process is simulated by lowering the virgin formation stresses (38 MPa) around inner holes to the initial reservoir pressure 24.6 MPa and the outer ones are kept to initial stress conditions after reservoir initialization.

It is noted that a plastic zone is developed as shown in Figure 6. This is due to the stress re-distribution around wellbore during the drilling process. It is critical to capture the developed plastic zones due to drilling and perforation, since the erosion coefficient $\lambda$ is linked to plastic shear strain as defined in Eq. (7). In other words, the larger the plastic shear strains are, the more intensive the erosion activity is. The plastic shear zones occur around the perforation tips, while the large shear stress forms and bridges all perforation tips together as shown in Figure 7.

On the contrary, there are no plastic shear zones developed in the case of openhole completion without perforation. A smaller shear stress ring as compared to the one in the perforated case develops as shown in Figure 8, with the formation being still in the elastic deformation regime.

4.2. Evolution of fluidized sand saturation

From this section on, we look at the field variable profiles due to drawdown in both completion cases. Figure 9 illustrates the distribution of the fluidized sand saturation $S_{fs}$ along the AB line in Figure 5 at five different times after drawdown. It is noticed...
that a sharp rise in fluidized sand saturation develops in the region near the perforations with the remaining part of the well being at near initial values of $S_{fso}$. The amplification factor for fluidized sand saturation near the perforation, defined as the current saturation value over the initial one, is about 40 times for time $t=0.11$ day, 250 times for time $t=0.41$ day, and 70 times for time $t=5$ days respectively. These numbers indicate that there is a dramatic increase in the creation of fluidized sand corresponding to sand production.

Figure 10 shows a similar trend in the openhole completion case. There is a sharp increase in sand production at the beginning followed by a gradual decline with time around the wellbore. This indicates that erosion activity subsides with time as there is no material left for the erosion to proceed around wellbore. As a comparison, spatial distribution profiles at time $t=0.41$ days are plotted in Figure 11&12 for both the perforated and openhole completion cases respectively.

4.3. Evolution of cavity propagation

Figure 13 shows the porosity history along line AB in Figure 5. In the perforated casing completion, the porosity around the wellbore is almost kept intact, while the porosity reaches 0.80 at the tips of perforations after 5 days drawdown, which indicates a cavity is being formed. Figure 15 further reveals that the erosion occurs intensively at the tips of perforations. The eroded zones gradually grow, and eventually merge together to form a large cavity. On the contrary, the maximum erosion activity does start simultaneously around the wellbore in the openhole completion case as shown in Figure 14. Figure 17 shows the coalescence of eroded zones into a ring of loose sand of about 0.2 m in radius.

The porosity values approach 0.77 and physically correspond to the formation of a cavity and
mechanical failure of the wellbore. Figure 18 shows a snapshot of the fully developed zone of high porosity. It is worth to mention that almost same size of cavity (0.7m in radius) is formed in both cases, but there is an almost intact region (0.3m in radius) at perforated casing case which can somehow tells us that wellbore might stay relatively stable from the mechanics point of view. This will be further investigated in later sections.

4.4. Evolution of pressure distribution
As the cavity enlarges, the permeability of the reservoir increases as it is a function of porosity in Eq.(9). The gradually increased permeability enhances the well productivity. As sand is being produced, the fluid pressure slowly depletes from initial values of 24.6 MPa on the outside boundary to 21.0 MPa around the wellbore, as shown in Figures 19 and 20.

4.5. Displacements, shear strain, and stresses
In this section, we look at the displacements, plastic shear strain and stress distributions in the well. Figure 21 shows x-direction displacement profile.
shown in Figure 22. This shows that openhole completion scheme will have large wellbore distortion subject to same drawdown condition. In most situations, the openhole completion most unlikely does not have any mechanical support (casing). Consequently, wellbore instability occurs more frequently in openhole completion due to sand production. Similarly, the pressure induced drag forces induce excessive plastic shear strains around perforations in both x- and y- directions (maximum value is about 20% after 5 days in Figure 23), while plastic shear strains of 30% develop around the wellbore in the openhole completion scheme as shown in Figure 24. It is also noted that the material strength parameters, i.e. cohesion $C$ and friction angle $\phi$ follow the same distribution as that of porosity since they are defined as a linear function of porosity in Eq.(12).

Considering the wellbore stability, it is very important to look at the stress re-distribution after sand production. Figures 25-26 show the distribution of effective stresses $\sigma_{xx}, \sigma_{yy}, \tau_{xy}$ at 5 days after drawdown. Due to fluid pressure reduction through three perforations, drag forces are imposed

---

**Fig. 19** Pore pressure distribution at selected times (perforated casing completion).

**Fig. 20** Pore pressure distribution at selected time (openhole completion).

---

**Fig. 21** Displacement in x-direction distribution at time $t=5$ days (perforated casing completion).

**Fig. 22** Displacement in x-direction distribution at time $t=5$ days (openhole completion).

**Fig. 23** Plastic shear strain distribution at time $t=5$ days (perforated casing completion).
upon three perforations, causing reduced stress values for $\sigma_{xx}$ and $\sigma_{yy}$ as shown in Figure 25. Figure 22 shows the tangential stress profile distribution. The high stress values indicate a highly sheared zone. In the openhole completion, the stress concentrated around wellbore. With the removal of formation materials, the maximum shear stress contours move into the reservoir, and form a damaged zone of about 0.4 m in radius.

Depending on the re-distribution of pore pressure and stress during erosion, the high shear stress zone grows, which in turn causes the evolution of plastic shear yielded zones.

4.6. Volumetric sand production and oil rates

In the previous sections, detailed spatial distributions of governing field variables with time were discussed and the analysis revealed local phenomena during sand production. From an engineering point of view, it would be interested in examining the total oil and volumetric sand production rates as integrated over the total perforation area $S$ ($P1$, $P2$, and $P3$) or around the wellbore (openhole completion). Hence,

$$q_{oil} = \int \int_S \|v_{f}\| dS; \quad q_{sand} = \int \int_{S_{fs}} \|v_{f}\| dS$$  \hspace{1cm} (13)

Figures 27 and 28 give both the oil and sand rates over the time of fluid drawdown. We observe that the sand production rate rapidly increases in an initial phase to reach a peak value in approximately 0.4 day. During this time period, the oil rate gradually increases as well. Then, this phase is followed by a decline in sand production rate.
corresponding to the decrease in availability of sand grains. However, the oil rate continues to increase given the enhancement in permeability of the reservoir induced by sand production. This trend is also observed in oilwells under sand production. Figures 29 and 30 show that the fluidized sand saturation increases at the beginning of production, then decreases as sand is being produced, while the porosity increases with time.

5. CONCLUSIONS

An oilwell study subject to different completion schemes (perforated casing and open-hole completion) is presented using a coupled reservoir-geomechanical model. The latter is based on an extension of a theoretical and numerical model that the authors have developed in the past to address the volumetric sand production and associated wellbore stability. This is done within the framework of mixture theory in which mechanics and transport equations are written for each of the concerned phases, i.e. solid, fluid (oil, water), gas, and fluidized solid. The numerical model is implemented into three integrated computational modules, i.e. erosion module, reservoir module, and geomechanics module. The key idea in the modular system is the reformulation of the stress-flow-erosion coupling so that any existing advanced stress and reservoir code can be incorporated with minimum development efforts. The system is powerful in terms of its capabilities, yet practical in terms of computer requirements. Depending on the complexity of addressed problems, different coupling methods can be used, i.e. decoupled, explicitly coupled, iteratively coupled, and fully coupled.

Numerical results show that the wellbore stability depends on the delicate interaction between geomechanics and hydromechanics processes. Formation tensile and plastic shear failures, incurred during hydrocarbon drawdown and in situ stress changes increase the sand production potential. In return, the production of sand also weakens the formation matrix through degradation of its mechanical strength (cohesion and friction angle). The self-adjusted mechanism enables the model to compute the volumetric sand production and cavity propagation in terms of fluidized sand saturation and porosity respectively. The results of two types of completion (open-hole and perforated casing completion) give an insight to guide the
design using different completion techniques and perforation patterns so as to optimize the hydrocarbon production.

6. ACKNOWLEDGEMENTS

The authors wish to express their sincere gratitude for funding provided by Alberta Ingenuity Fund (AIF) and the National Science and Engineering Research Council of Canada (NSERC).

REFERENCES


